

TASKS INVOLVING MODELS, TOOLS AND REPRESENTATIONS:

Making the Mathematics Explicit as We Build Tasks into Lessons



BARBARA CLARKE and

PETER SANDERS

describe Type 1 tasks
and discuss how
they contribute to
mathematics learning.



Overview

With teachers and students in Government and Catholic schools in three geographical clusters in Victoria, the Task Type and Mathematics Learning (TTML) project is investigating the best ways to use different types of mathematics tasks, particularly in Grades 5 to 8. The focus of our research is to describe in detail how the various task types contribute to mathematics learning, the features of successful exemplars of each type, constraints that teachers might experience and teacher actions that can best support students' learning. This article illustrates the process used early in the project in one of the clusters in which they used the Japanese lesson study model to explore the opportunities and constraints of using classroom tasks.

What is a Type 1 task?

Type 1 tasks involve the introduction to, or use of models, representations, tools, or explanations that elaborate or exemplify the mathematics. There is no attempt to link mathematics to its practical applications. Following student work on the task, the teacher leads a discussion on the mathematics that has emerged from the task, and seeks to draw out commonalities and generalisations.

Such tasks are associated with good traditional mathematics teaching (see Watson & Mason, 1998). The teacher commences with an important mathematical idea and proposes tasks that involve models or representations or tools, which help students to understand the mathematics. For example, consider the Maze Game, in which students move a counter around a game board, making choices between operations such as “+1.5”, “ $\times 0.8$ ”, “ $\div 0.5$ ” etc., with a view to increasing their score at each stage. In using this game, the teacher has a very clear mathematical intent: to confront the widespread misconceptions that multiplication always yields a larger answer and division always yields a smaller answer.

An example of a Type 1 task: Colour-in Fractions

Type 1 tasks often involve a game to engage students, but always with a clear mathematical focus. Colour-in Fractions uses a representation (the fraction wall) in a game context. The set-up and rules are given below.

Colour-in Fractions

Students have two dice that when thrown and combined as a pair, create fractions up to twelfths, and a fraction wall. They colour in sections of the wall that correspond to the fractions that they roll with the dice.

- one die labelled 1, 2, 2, 3, 3, 4 in one colour (the numerator)
- another die labelled $1/2$, $1/3$, $1/4$, $1/6$, $1/8$, $1/12$ in another colour (the denominator)

The wall is like this:



Players in turn throw both dice. They make a fraction, the first die being the

numerator. Each line is one whole.

They then colour the equivalent of the fraction shown. For example, if they throw 2 and $1/4$, then they can colour in:

$2/4$ of one line or $4/8$ of one line or $1/4$ of one line and $2/8$ of another or any other combination that is the same as $2/4$.

If a player is unable to use their turn, they “pass.” The first player who colours in their whole wall is the winner.

There can be further mathematical questions posed based on the game and class discussion of strategies.

The teacher is likely to have a clear mathematical intent in using this game with a particular focus on developing students' understanding of equivalence, improper fractions, and addition of fractions with the “fraction wall” model as a key component. That being said, different students may draw quite different learning from the same game. One student may discover for the first time that $2/3$ is equivalent to $8/12$ and why, while another may determine that when left with $1/12$ to fill at the end of the game, there is only one chance in 36 of a successful roll each time.

The importance of the model and the explicit focus on the mathematics are features of this task and the fraction wall has the potential to provide an ongoing tool for the student to use in other situations. It should be noted that:

The aim of learning with a model is to give a student a tool to think with; something that they can draw upon to interpret symbolic work. So models should be carefully chosen and used thoroughly and consistently for some time (State of Victoria, 2007).

Teachers from the project, in characterising Type 1 tasks, highlighted the importance of linking the model or tool explicitly and directly to the mathematical concept.

Although these tasks are not contextualised, there is sometimes a “hook” that helps to engage the students. One example is the Chocolate Fraction activity

(see Clarke, 2006, for a full explanation of the task) where sharing of chocolate represents an engaging context but also a model for the development of the concept of fraction as division.

The role of explanation

In considering the role of teacher explanation in Type 1 tasks, it is not a necessary feature that exposition is required. The provision of the model or representation can enable the students to generate the mathematical ideas and justification. This is clearly the case in the Colour-in Fractions task. When this was trialled, the teachers were able to use the thinking and explanation of ‘selected’ children to develop the mathematical concept. Students were able to generate their own explanations, which the teachers then used to make explicit the mathematical concepts.

With a ‘good’ Type 1 task, the mathematics is inherent in the model or representation and there seemed to be limited teacher explanation required. Some sort of introduction was given with a focus on tuning in to the mathematics and reviewing of some prerequisite knowledge.

The mathematical purpose is pivotal in Type 1 tasks and, as a result, it influences the teachers’ decision making. Arguably there is likely to be less opportunity for a detour from the plan than with other task types, but also the teacher may be less willing to deviate into a different area of mathematics, even when faced with an opportunity, given the clear mathematical intent behind the chosen task.

Advantages and challenges identified by teachers using Type 1 tasks

Teachers were asked to comment on the advantages which had emerged in their use of these kinds of tasks. The prompt was: “What do you see as the advantages of using this task

type in your teaching?” The following are representative responses:

- Gives hands on experiences and aids children’s understanding.
- Use of models helps with the explanation and gives students the opportunity to experiment with practical materials for better understanding.
- Great for visual learners. Very ‘hands on’ and logical approach to teaching maths. It gives the children an excellent tool to assist in their explanations of why/how they did something.
- The model focusses the students on the key mathematical ideas of the task/lesson, teacher explanation and probing of ideas of the students’ explanations enables them to engage with the key ideas.

A number of teachers saw the value in these tasks for developing student understanding. There were also a number who commented on the willingness of students both to participate and to engage with the mathematics.

Another prompt for teachers was, “What makes teaching using this task type difficult? What are the challenges in using this type of task?” The following responses were representative:

- Organisation and provision of all materials needed.
- The time taken to plan and organise a ‘good’ lesson.
- Finding appropriate activities.
- Clarity of the model/exemplar. Extending the model/exemplar into a lesson with meaningful independent/group work.

Some of these challenges appear to be related directly to the task type, including the time required to prepare the materials. The broader challenge of taking the task or idea and transforming it into a lesson was also identified.

Turning a task into a lesson: Stories of successful collaborative practice

Some of the constraints teachers identified above, particularly those related to organising a ‘good’ lesson and extending the model into a successful lesson, have caused problems for teachers working independently in their own schools. The Berwick South teachers decided to deal with these difficulties by working collaboratively across the cluster with groups of teachers working together to design ‘exemplary’ Task Type 1 lessons.

They chose a professional learning model, Lesson Study, to help in this process. Lesson Study is a model recommended by the Victorian Department of Education and Early Childhood Development (State of Victoria, 2005) and has been advocated as a successful model for teacher professional development (Hollingsworth & Oliver, 2005; White & Southwell, 2003).

The fifteen teachers in the cluster were divided into five groups of three and each group met for a full day to plan, teach, reflect upon, and refine lessons. Planning and designing the lesson took place in the morning until recess. One teacher from the group taught the lesson between recess and lunch with the others observing, and the afternoon allowed the opportunity for thorough evaluation of the taught lesson including revisions to the lesson plan and individual teacher reflections. A number of successful Task Type 1 activities have been turned into successful lessons using this model.

Additionally, teachers gained useful insights into their own mathematics teaching practice, and this aspect as well as the success of Lesson Study is apparent in these reflections from teachers who were involved.

- I thought that allocating a whole day to this experience was a fantastic idea. It meant that we could complete the structure of the lesson study in its entirety and make it a really worthwhile experience. The set-up of the day also worked really well. It was great to get

together with teachers from another school to plan a ‘perfect’ lesson on a particular concept with a set focus. This emphasised different planning and teaching techniques and also gave us an opportunity to discuss the difference between our groups of children.

- Working with teachers from other schools is invaluable. We get so familiar with the processes and habits of our own school, it is fantastic to share ideas and strategies across schools and get a different perspective.

One of the constraints referred to earlier concerned having adequate time to plan and organise a ‘good’ lesson. The teacher comments below indicate that Lesson Study was a useful way of addressing this issue:

- The lesson study provided an opportunity to discuss with other teachers the process of planning and implementing a specific lesson, which is not something that we often (or ever!) have time to do as individual teachers.
- I felt planning the lessons together was very powerful.

Teachers also found the other two components of Lesson Study very valuable, namely, the teaching of the lesson and the reflection afterwards.

- It was fantastic to put aside the whole middle session to actually observe the lesson being put into practise. I liked the inclusion of different tasks for observers such as observing teacher questioning and children’s responses.
- Observing how lessons can completely change their course but still be successful in teaching a concept.
- The reflection session in the afternoon proved very valuable. We were able to discuss which aspects of the lesson worked well and which parts we would need to modify in order to make the lesson even more successful. It was interesting to hear from the different observers and listen to what they found out in their particular observational role.

Lessons from turning tasks into lessons

Other difficulties associated with Task Type 1 have also been addressed through the Lesson Study process. A number of teachers were concerned in the early stages as to exactly what these tasks looked like and the clarity of the model/exemplar. Some responses following Lesson Study indicate the development of teachers thinking about this task type:

- It took a while for me to get the idea of Task Type 1. After seeing it being taught in my grade it confirmed my teaching was on the right track.
- The types of lessons developed has helped me understand Task Type 1 better. They are beginning to merge for me. Lesson Study was very helpful here.

The final comment below alludes to a powerful aspect of this project, namely, that the analysis of different task types is influencing teachers' approaches to mathematics teaching and learning in their own classrooms.

- Realising that I need to integrate more Task Type 1 activities into my teaching program and doing so with great success.

While from a research point of view, useful insights are being gained into the processes involved in each task type such as the insights into Task Type 1 presented in this article, from the teachers' perspectives, their usual approach to teaching in their own classroom is being challenged. However, the addition of the professional development component through Lesson Study has enabled these challenges to be supported and changes successfully implemented. This highlights the key importance of ensuring research

in mathematics education involves teachers working in their own classrooms and is proving to be a valuable component of the Task Types and Mathematics Learning project.

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Barbara Clarke
Monash University
<barbara.clarke@education.monash.edu.au>
Peter Sanders
La Trobe University

APMC